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HYPERSONIC FLIGHT AND THE RE-ENTRY PROBLEM

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INTRODUCTION

Progress in transportation has been brought about more by revolutionary than by evolutionary changes in methods of propulsion. Over many centuries of use, sailing ships were greatly improved, yet they gave way to the steamship less than one hundred years after the invention of this new means of propulsion. In less than 20 years the horse-drawn carriage was replaced by the automobile, and more recently, in this country, the steam locomotive has submitted to the Diesel engine.

Although these changes have often resulted in more economical transportation, they have not always done so. Speed improvement has also been an impetus. True, speed and economy often go hand-in-hand, but speed alone may constitute the sole reason for change, particularly when the vehicle has military usefulness. In the history of development of the airplane, speed has played a particularly prominent role.

At its inception, the airplane could hardly have been considered useful, militarily or otherwise. With its engine which weighed 17 pounds per horsepower, the first Wright brothers machine could barely fly. The initial success, in fact, is attributable to the knowledge and ingenuity of the inventors.

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They knew the vital need for providing their craft with a sufficiently great wing span to minimize the induced drag penalty imposed by the heavy engine. They then devised a biplane configuration which permitted the required long span to be realized with a minimum of structural weight. In spite of the marginal performance of the first Wright machine, the situation was clearly correctable. Charles Manly had already constructed, for the ill-fated Langley Aerodrome, an engine no heavier than the 12-horsepower Wright engine which had produced, in three 10-hour tests, more than 50 horsepower - a most remarkable achievement.

To the sagacious, no doubt, while the general future of the airplane seemed bright, the military future seemed brilliant. Although the first Wright airplane had only reached 31 miles per hour, it was evident that high speed would be a particular virtue of the aircraft to come. This high speed would be unusually significant for the military airplane since it would provide an invulnerability to possible counterweapons. Progress in improvement was sufficient in the next 10 years to enable the airplane to play a supporting role in the First World War, and, in the following 20 years to give it a decisive role in the Second World War. In spite of the fact that over this whole period the performance gains had been truly amazing, it then appeared that further significant increase in speed was unlikely. One cause was traceable to the propeller. As the aircraft speed had increased, the tip speed of the propeller had approached too close to the speed of pressure propagation in air. Thus compression shock losses occurred which seriously decreased the attainable lift-drag ratio of the blades, and the propulsive efficiency dropped accordingly. The use of thinner blade sections helped to delay the onset of the difficulty. However, this cure clearly was short-lived at best. Even presuming that the propeller difficulties could be circumvented, the increase in airplane drag to be anticipated at transonic and higher speeds would require such a large increase in power as to make the propulsion system again excessively heavy. The propeller-driven airplane powered by the piston engine had reached an impasse.

A drastic change was needed and it came. Before the Second World War was finished, Sir Frank Whittle in England and engineers at the Junkers Company in Germany had developed turbojet engines that gave promise of providing the required high power at much lower weight than could be attained with a piston-engine and propeller combination. In the years since that war the promised performance of the turbojet engine has been realized. For the turbojet, higher speeds are not only desirable but necessary if high propulsive efficiency is to be achieved. It fell to the aerodynamicist to make the attainment of efficient transonic and, later, supersonic flight a reality. Thin wings of low aspect ratio provided one solution for reducing drag; swept wings, another; and means for promoting favorable interference between aircraft components, a third. The aerodynamic improvements, in general, brought new and severe structural problems. Nevertheless, in time the revolution was complete. The new breed of airplane did not provide the increased speed without penalty to range, for the aerodynamic efficiency had suffered a permanent setback from the wave drag which had been incurred on entering the supersonic regime. By careful aerodynamic design, however, the penalty incurred due to wave drag could be kept within acceptable bounds. The effect on range of the unavoidable decrease in lift-drag ratio could be minimized by the use of more efficient structures to provide a more favorable ratio of gross weight to empty weight.

Now that the sonic speed hurdle has been passed and truly supersonic flight has become commonplace, the quest for more speed continues, and it devolves upon the power plant to produce the increased thrust required for higher speed with a minimum increase in engine weight. Two engine types have received consideration in this regard. One is the ramjet which is a natural progression from the turbojet or turbojet with afterburner. The other is the rocket motor.

Fig. 1 shows the weights of the several power plants per unit thrust horsepower as a function of flight speed. The attractiveness of the rocket motor from this standpoint is obvious. It has a second advantage in having no maximum limit on speed as do the air-breathing engines. The limit of usefulness for the piston-engine-propeller drive occurs when the speed is sufficiently high that the turbojet engine performs about as efficiently but with less weight. A similar limit occurs for the turbojet since at a sufficiently high speed (Mach number of the order of 4) the compressor-turbine serves only to decrease the efficiency and increase the weight in comparison with the ramjet. For the ramjet the limit of usefulness as a heat engine occurs when, due to the heat of compression, the temperature of the air entering the combustor reaches the temperature for chemical equilibrium through combustion so no heat can be added. The problems of air-breathing engines have been[1] and are now the subject of much research. The limit speeds will certainly be increased but probably not indefinitely. A third advantage of the rocket motor is that since it does not require oxygen from the atmosphere it is not altitude limited. It performs best, in fact, *in vacuo*.

In spite of these advantages the rocket motor has the important disadvantage that it requires all of the chemicals needed for its operation to be carried aboard the vehicle it powers, while the other engines require only fuel. Fig. 2 shows the specific impulses in pounds thrust per pound per second of propellant for the several engines as a function of speed, and indicates the very inferior position of chemical rocket motors in this regard.

Clearly for high-speed short-range flight the rocket's advantage of light engine weight far outweighs its disadvantage of low specific impulse. However, for high-speed longer range flight the situation is not so obvious. It is the purpose in this paper to discuss such questions as: Can rocket vehicles compete with supersonic airplanes on an efficiency basis for long-range flight? What types of rocket vehicles, if any, appear attractive and under what circumstances? What new problems occur with these vehicles and do they appear surmountable?

PERFORMANCE

There are three types of long-range, high-speed vehicles which appear to be of particular interest; the ballistic, the glide, and the skip rocket. The typical flight trajectories of these rockets are shown in Fig. 3. The ballistic trajectory is so well known that no discussion of it is needed, but it is perhaps desirable to discuss the other two briefly.

The glide rocket is boosted by the rocket motor to an altitude and speed such that at the end of boost the dynamic pressure is that required for the vehicle to fly without power at some given lift coefficient. This altitude, which gradually decreases as the vehicle loses momentum due to drag, has been termed by Sänger[2,3] the "equilibrium altitude." The aerodynamic lift required for flight is the weight less the centrifugal force resulting from the curved flight around the earth. Thus the aerodynamic lift, which equals the weight for low speed, becomes zero as satellite speed is approached. The equilibrium altitude as a function of speed, therefore, varies as shown in Fig. 4 for a range of wing loadings from 10 to 100 pounds per square foot, and for a lift coefficient of one-tenth. It is at first, I think, a little surprising that even for speeds closely approaching satellite speed the equilibrium altitudes remain below 250,000 feet. The glide path relative to the earth below it, therefore, remains very nearly flat at all supersonic speeds.

The skip-rocket trajectory is composed of a succession of ballistic paths each connected to the next by a "skipping phase" during which the vehicle enters the atmosphere, negotiates a turn at some given lift coefficient, and is then ejected from the atmosphere. In each skip the minimum altitude must be less, of course, than the equilibrium altitude at which the glide rocket would fly at the same lift coefficient and speed, since an increased lift is required to execute the turn.

For all three rocket vehicles we will be concerned only with those trajectories which yield the maximum range for a given energy input.[4]

For the glide rocket and for the skip rocket when in the atmosphere the lift coefficient for maximum lift-drag ratio must be maintained to achieve this end. For the glide rocket, as noted earlier, the flight path angle during glide is the small angle required to maintain equilibrium altitude as speed decreases during flight. For the ballistic rocket the least-energy trajectory requires, as a function of range, the flight path angles at end of boost shown in Fig. 5. For the skip rocket having a very low lift-drag ratio the optimum angle at end of boost approaches that for the ballistic rocket. As the lift-drag ratio increases, the optimum angle will be less than that for the ballistic vehicle except at the longest ranges. This case is indicated in Fig. 5 for a skip rocket having a lift-drag ratio of 6.

It is a purpose of this paper not only to compare optimum performance of these three rocket vehicles with one another, but also with that of a typical supersonic airplane powered by air-breathing engines. The efficiency of flight is perhaps best measured by the cost of delivering a given payload a given distance - the higher the cost the lower the efficiency. It is clearly beyond the scope of this paper to actually compute this cost. Rather I will use as a measure of the efficiency the ratio of initial weight to payload. All the components which go to make up the initial weight (fuel, structure, engines, etc.) do not have the same unit cost nor, for a nonexpendable vehicle, is all this material wasted. Nevertheless, this ratio should provide a fair estimate of efficiency, particularly if the vehicle is considered expendable.

In the following, I will first discuss the ratio of initial weight to final ("empty") weight and then the ratio of final weight to payload so that lastly some reasonably intelligent estimates, I hope, can be made of the relative flight efficiency.

For the aeronautical engineer the ratio of initial weight to final weight, and the range, are related by the Breguet equation. One form of this equation is

$$R = \left(\frac{L}{D}\right) IV \ ln \frac{W_i}{W_f} \qquad (1)$$

wherein

$\frac{L}{D}$ lift-drag ratio

W_i take-off (initial) weight

W_f landing (final) weight

I specific impulse of the fuel

V flight speed

The product IV is equal, of course, to the thermal propulsive efficiency times the heat value of a unit weight of fuel.

For the comparison of the several vehicles it would clearly be desirable to develop for rockets an equation corresponding to the Breguet equation. This was done in reference 4 in the following way. First, it should be noted that the speed at end of rocket boost can always be written

$$V_b = I_e g \ ln \frac{W_i}{W_f} \qquad (2)$$

where

g   acceleration of gravity

I_e effective specific impulse

The effective specific impulse would be the actual specific impulse of the propellant if the thrust were extremely large compared to the weight, and aerodynamic drag during boost were negligible. These conditions are not met in practice so the effective specific impulse must always be less than the actual, but for efficient designs the difference is not great.

Next we define an "effective drag" force in unpowered flight which, when multiplied by the range, equals the kinetic energy of the rocket vehicle; that is

$$D_e R = \frac{W_f}{2g} V_b^2 \tag{3}$$

and an "effective lift" equal to the final weight

$$L_e = W_f \tag{4}$$

From Eqs. (3) and (4) it follows that

$$R = \left(\frac{L}{D}\right)_e \frac{V_b^2}{2g} \tag{5}$$

From Eqs. (2) and (5), then, a rocket equation similar in form to Breguet's equation is obtained

$$R = \left(\frac{L}{D}\right)_e I_e V_e \ ln \frac{W_i}{W_f} \tag{6}$$

wherein the effective velocity is

$$V_e = \frac{V_b}{2} \tag{7}$$

With Eqs. (1) and (6) we will be able to make the first comparisons we desire for it is the product

$$\left(\frac{L}{D}\right) IV$$

which constitutes our standard of excellence.

The power-plant characteristics of Fig. 2 may be replotted as $I_e V_e$ (which for airplane propulsion is IV) as a function of flight speed. These products are shown in Fig. 6. It is clear that this "propulsion product" is very low generally for the rocketcraft, for only at the highest flight speeds are they at all comparable with the air-breathing engines. This, I think, is the dismal picture that has too often brought on the usual rejection of rocket craft as reasonably efficient vehicles. However, it is not alone this propulsion product which determines the efficiency since it is also necessary to consider the effect of lift-drag ratio on the range-weight relation.

The effective lift-drag ratios for rockets have been determined in reference 4. In Fig. 7(a) are shown the effective lift-drag ratios for the ballistic rocket, and for the glide and skip rockets when the maximum aerodynamic lift-drag ratio has the low value of 2. The skip vehicle is best and, oddly enough, the glider is barely better than the ballistic rocket. In Fig. 7(b) is shown the effective lift-drag ratio for the ballistic rocket, again, and for the glide and skip rockets having the moderately high aerodynamic lift-drag ratio of 6. Here, the glide and skip vehicles are very nearly equal and, of course, the ballistic vehicle is much inferior. The most striking and important feature to be noted in both these figures is that for all the rocket craft the effective lift-drag ratio continuously increases with increasing range. One physical explanation for this increase for the ballistic rocket is the following: When the range is half the circumference of the earth, the speed at end of boost is required to be just satellite speed and for greater range no increase in energy input is required. Thus the effective drag, by definition, continuously decreases with increasing range. For the glide rocket a physical explanation that may perhaps be more apparent is that, as speed is increased to obtain longer range, the centrifugal force increases. Thus less of the weight must be supported by aerodynamic lift so that the aerodynamic drag is less.

If one now combines the results of Figs. 6 and 7, the ratio of initial-to-final weight can be obtained. These ratios are shown in Figs. 8(a) and 8(b) for the rockets and the airplane. The assumptions, here, are that the effective specific impulse for the rockets is a presently obtainable value (300) in Fig. 8(a) and for twice this value (600), which might be obtainable in the future, in Fig. 8(b). It is also assumed that the IV product for the air-breathing, hydrocarbon-burning engine is slightly more than 800 nautical miles (see Fig. 6) and the aerodynamic lift-drag ratio for the airplane, the glide rocket, and the skip rocket is 6.

It is seen that when the range is sufficiently great, the improvement in effective lift-drag ratio offsets the disadvantageous propulsion characteristics of the rocket so that the rocket vehicles on the basis of weight ratio become competitive with the airplane.

It is next in order to consider the component weights, other than payload, that go to make up the final weight. As regards the propulsion system weight, the advantage here is, as we have seen, with the rocketcraft. For the structural weight the situation is not so clear. The ballistic vehicle would appear to have the advantage since it has no wings; however, it will tend to have the largest tankage weight. Consideration must also be given to the fact that it may experience large aerodynamic loads on entering the atmosphere which may adversely affect structural weight. The skip rocket will be similarly affected. A factor of primary significance for all of the long-range rocketcraft is that they are subject to intense aerodynamic heating as a result of the high speeds attained. Even if the heat can be radiated away, the high surface temperature must adversely affect structural weight. If the heat cannot all be radiated, then the final weight must be increased by the required weight of coolant needed to protect the vehicle. It is fitting, then, to discuss in some detail several of the more important factors which influence final weight in an effort to gain insight into the relative ratios of final weight to payload.

Aerodynamic Heating

Before discussing the detailed heating problems associated with each of the rocket types it is well to review the nature of the problem from a general point of view. First, as indicated in Fig. 9, for long-range rockets the speeds at rocket burnout are generally 10,000 feet per second or greater. In the usual case, the speed at landing will be very small compared to burnout speed so that virtually all of the kinetic energy imparted to these craft must appear as heat. The heat equivalent per pound of weight as a function of flight speed is shown in Fig. 10. Also shown is the heat required to convert one pound of ice to steam at 1000° Fahrenheit as a "measuring stick" of what a fairly good coolant can handle. It is evident that if all of the kinetic energy appeared as heat within the vehicle and if the time rate of heat addition were so great that little of this heat could be radiated away, the problem might well be insurmountable even for some cooling system far better than this ice-to-steam system. To make this point clear, consider a ballistic vehicle having a flight range of 1500 nautical miles which requires (Fig. 9) a speed at end of boost of something over 15,000 ft/sec. If the coolant to absorb the kinetic energy were three times as effective as the ice, all of the final weight would be in coolant (see Fig. 10) so that no payload could be carried. A practical vehicle could obviously not be built if the coolant weight required became even a large fraction of the final weight.

How can this situation be avoided? One of two solutions to the problem may be applied. If the rate of heat input is extremely high so that but a small part of the heat convected to the vehicle can be radiated away, then it is mandatory that a minimum fraction of the total kinetic energy appear as heat within the vehicle. On the other hand, if an excessively large fraction of the kinetic energy must be convected as heat to the vehicle, the time rate of this convective heating must be sufficiently slow that a large fraction of the heat can be radiated away at a surface temperature that is structurally permissible. Let us now consider, in light of the above, the heating problems of the ballistic, glide, and skip vehicles.

The ballistic vehicle requires the first method of solution for the heating problem since the convective heat input rates will usually be several orders of magnitude greater than the rate at which surfaces at temperatures near the melting point of metals can radiate. Fortunately,[5] the total convective heat input can be kept satisfactorily low since the fraction of the total kinetic energy change which must be accepted as convected heat to the vehicle is approximately

$$\frac{C_f S}{2 C_D A}$$

wherein

C_f frictional drag coefficient per unit of the wetted area S

C_D total drag coefficient based on the reference area A

That is to say by making the friction drag small compared to the total drag, a large fraction of the heat developed is given to the atmosphere (wasted in shock waves, etc.), and the heat convected to the vehicle is kept small. Thus for ballistic vehicles the solution to the heating problem is to employ blunt shapes which have high pressure-drag coefficients.* In this way the heat convected can be kept to a percent or so of the kinetic energy change so that the weight of coolant required may be correspondingly small.

For the glide vehicle, this solution of the heating problem is not possible since, for the glider to be superior to the ballistic vehicle, it must develop a high aerodynamic lift-to-drag ratio which is incompatible with a small ratio of frictional to total drag. However, the glider gradually converts its kinetic energy into range so that, unlike the ballistic vehicle, the time rate of convective heating is not large, as has been

*This idea has no doubt occurred to many. It has been brought to my attention, for example, that Dr. H. H. Nininger, Director of the American Meteorite Museum, had suggested the possible advantage of blunt shapes for missile re-entry bodies as a result of examination of the shape and surface condition of many metallic meteorites.

shown.[4] Hence for the glider there is a possibility of radiating all or nearly all the convected heat with surface temperatures which are structurally permissible. Eggers has shown, as indicated in Fig. 11 taken from reference 6, that the maximum temperature for radiation equilibrium of an average surface element for a conical glide vehicle could be about 1600° F which is within that allowable for some presently available materials. The radiation equilibrium temperature is adversely affected by increasing the wing loading and accordingly satisfactory values of loading will generally be low ones. Moreover, while the use of coolants at such "hot spots" as the bow of the body or wing leading edges, when present, would probably be required,[6,7] the weight of the coolant should not necessarily be excessive.

For the skip rocket, the heating problem appears to be much more formidable. As with the glider, high lift-drag ratios must be developed if it is to be a useful type so that a low ratio of frictional to total drag cannot be realized. A large fraction of the kinetic energy change in the first skip, which is, in itself, a fairly large fraction of the total kinetic energy at burnout, must be convected to the vehicle. On the other hand, the time spent in the atmosphere is so small during this skip that the rate of convective heating is high. Eggers,[6] again, has indicated that a conical skip rocket during the first skip would reach the very high radiation equilibrium temperatures shown in Fig. 11. These temperatures would require the extensive use of coolant. Thus the weight penalty would probably be so excessive as to rule out this vehicle as impractical or even impossible except for short-range flights.

Aerodynamic Loads

The effects of aerodynamic loads on rocketcraft are more complex than they are for more conventional aircraft since when loads are high aerodynamic heating is usually intense. Thus rocket structures when subjected to high stress due, say, to bending moments may simultaneously be subjected to additional localized stress resulting from severe temperature differences within the structure. For hypersonic vehicles, in fact, thermal stress may easily be a principal cause for structural failure as is evidenced by the explosive spalling types of failures which, fortunately for us, occur with many meteors.[8]

By astute choice of materials and ingenuity in arrangement, however, the designer of a rocketcraft can keep these thermal stresses from reaching untenable values. Much work is, of course, in progress by structural engineers to find solutions to these thermal stress problems[9] and much more will be required before the same confidence in design can be provided that has been attained in the conventional structural design of airplanes. The time well may come when our knowledge of these thermal aspects will have advanced to the degree that only small penalty in weight will have to be paid for these complicating effects. On the other hand, the direct aerodynamic load will always be a vital factor in determining the weight of structure, and it is about these loads that we will be concerned here.

Since the aerodynamic loads are directly proportional to the dynamic pressure and hence the air density, it is well to note that the density variation in the atmosphere, over the range which is important from the load standpoint,[4,5,10] can be approximated by the exponential relation

$$\rho = \rho_o e^{-\beta y}$$

where ρ_o is sea-level density, β a constant, and y the altitude.

This particular functional relationship leads to some singularly interesting simplifications.[5] The variations with altitude of the deceleration experienced by a ballistic missile in its flight down through the atmosphere is a case in point. The velocity can be expressed as

$$V = V_E e^{-\frac{C_D \rho_o A g}{2\beta W \sin \theta_E} e^{-\beta y}}$$

where

θ_E trajectory angle below the horizontal

V_E velocity on entrance to the atmosphere

C_D drag coefficient

g acceleration of gravity

W weight

A the reference area on which the drag coefficient is based

and the deceleration in terms of g is

$$\frac{dV/dt}{g} = - \frac{C_D \rho_o A V_E^2}{2W} e^{-\beta y} e^{-\frac{C_D \rho_o A g}{\beta W \sin \theta_E} e^{-\beta y}}$$

If the speed at impact is less than about 61 percent of entrance speed, which will usually be the case, then it can be shown that the maximum deceleration is simply

$$\left(\frac{dV/dt}{g}\right)_{max} = -\frac{\beta V_E^2 \sin\theta_E}{2ge}$$

where e is the Naperian logarithm base. That is to say, the maximum deceleration that can be experienced is independent of the drag coefficient and, hence, of the vehicle's shape. This deceleration is also independent of the weight.

As an arbitrary example consider a series of solid iron spheres which have diameters ranging from 1/100 foot to 10 feet and for which the entrance velocity is 20,000 feet per second and flight path is vertical ($\theta_E = 90^\circ$). These spheres range in weight from less than 2 grains to over 100 tons. The decelerations for each is shown in Fig. 12. It is seen that not only is the maximum deceleration the same for all, but the functional relation of the deceleration with altitude is also identical. The deceleration curve is simply shifted to higher altitudes the lighter the sphere. Moreover, even for the heaviest sphere the maximum deceleration is reached prior to impact, so it is to be expected that for any ballistic vehicle which employs a high-drag shape to minimize the heating problem the maximum deceleration will similarly be reached before impact. It is also important to note that for each of the iron spheres the decelerations are large compared to gravity over a range of altitude of about 100,000 feet. This is also the range over which the heating rates are high.

For a ballistic vehicle which for a given range has the least kinetic energy at rocket burnout, both the velocity and trajectory are determined so that the deceleration can be calculated. This deceleration is given in Fig. 13. It is seen that the worst deceleration occurs for a range of 4,000 nautical miles when it is something less than 60 times gravitational acceleration. The maximum deceleration falls off with increasing range in spite of the fact that the burnout speed is increasing because the angle of approach to the earth becomes more flattened and so the time rate of density change is decreasing enough to more than offset the speed increase.

In any event, for the nose portion of a ballistic vehicle which is of bluff shape, necessitated by aerodynamic heating considerations, the decelerations shown in Fig. 13 are not so high as to seriously increase the structural weight. In fact for ranges of half the earth circumference and greater, the decelerations are within human tolerance so that return of a manned satellite in the form of a ballistic-type vehicle seems reasonable[6] even when aerodynamic lift is not employed to minimize the re-entry forces experienced.[6,11]

For the glide rocket, the aerodynamic load problem is, in the main, the same as that for the conventional airplane with the exception that we must deal, in whole or in part, with a hot structure. Moreover, the gust load problem is as yet not well defined.

For the skip rocket the aerodynamic loads, in contrast with aerodynamic heating, do not appear to present too severe a problem. The normal accelerations which occur in the first skip are shown as a function of total range in Fig. 14. The aerodynamic lift-drag ratio assumed is 6. It is seen that these normal accelerations generally are of the order we have become accustomed to in fighter airplane design. Even aside from aerodynamic heating consideration, however, the structural problem is far more serious than for the glide rocket.

Stability

The aerodynamic stability of rocketcraft presents some unusual problems which can importantly influence both structure and guidance.

For the ballistic missile, Friedrich and Dore[12] have given a general method for the analysis of the stability. In reference 13, this method was used with certain simplifying assumptions to describe the oscillatory behavior of a ballistic body at supersonic speeds. The general solution for the angle of attack, α, is

$$\alpha = e^{k_1 e^{-\beta y}}\left[C_1 J_o\left(2\sqrt{k_2}\, e^{-\frac{\beta y}{2}}\right) + C_2 Y_o\left(2\sqrt{k_2}\, e^{-\frac{\beta y}{2}}\right)\right]$$

In this equation $J_o(-)$ and $Y_o(-)$ are the zero-order Bessel functions of the first and second kinds, respectively. The "dynamic stability" factor is

$$k_1 = \frac{g\rho_o A}{4\beta W \sin\theta_E}\left[C_D - C_{L_\alpha} + \left(C_{m_q} + C_{m_{\dot\alpha}}\right)\left(\frac{l}{\sigma}\right)^2\right]$$

while the "static stability" factor is

$$k_2 = -\frac{g\rho_o A}{2\beta^2 W l \sin^2\theta_E}\left[C_{m_\alpha}\left(\frac{l}{\sigma}\right)^2\right]$$

and

C_D the drag coefficient

C_{L_α} the rate of change of lift coefficient with angle of attack

C_{m_q} the rate of change of moment coefficient with angular velocity

$C_{m_{\dot{\alpha}}}$ the rate of change of moment coefficient with time rate of change of angle of attack

C_{m_α} rate of change of moment coefficient with angle of attack

$\frac{l}{\sigma}$ ratio of the characteristic length to the radius of gyration

C_1, C_2 constants of integration

and the other symbols are as previously defined.

If the body on entering the atmosphere has its axis misaligned by an angle α_E with respect to the flight path but has no angular velocity, then C_2 is zero and the solution becomes

$$\frac{\alpha}{\alpha_E} = e^{k_1 e^{-\beta y}} J_o\left(2\sqrt{k_2}\, e^{-\frac{\beta y}{2}}\right)$$

Then to illustrate the typical oscillatory behavior, suppose k_2 has a value of 10^5, which is a likely one, and k_1 has in turn the values -10 (damped), 0, +10 (undamped). The angle-of-attack variations with altitude would then be those shown in Fig. 15. If we look first at the oscillations when the dynamic stability factor is zero, it will be seen that during the descent as the air density increases the missile responds by pitching about zero angle with decreasing amplitude. The motion, thus, is a damped one, as seen by the envelope curve, but not damped in the usual sense that energy has been removed from the system. Rather, this behavior is akin to the motion that would occur with an oscillating mass on a spring if the spring constant were to increase continually. The effect of the dynamic stability factor on the amplitude, in fact, only becomes significant in decreasing ($k_1 = -10$) or increasing ($k_1 = +10$) the amplitude at altitudes below about 100,000 feet where the amplitude has already been reduced to one-tenth the original value.

The effect of the oscillations that occur must be allowed for in the design of whatever cooling system is used to protect the vehicle from the aerodynamic heating that is experienced, since the local heat input rate will be changed by the motion from what they would be for the case when α is zero.

The frequency can also be determined by the method of reference 13. It increases with decreasing altitude until the altitude for maximum deceleration is reached (i.e., where the velocity is 61 percent of the entrance speed) and then decreases. The frequency, in a typical case, may easily become as high as 10 cycles per second and hence may introduce important stresses due to inertial loads which must be considered in the structural design. Proper orientation of the vehicle by reaction controls prior to the entrance to the atmosphere can, of course, prevent the occurrence of such additional problems.

For the long-range glide rocket which properly follows the "equilibrium" trajectory, the stability problems are generally those for conventional airplanes except that at the highest speeds, when a sizable part of the vehicle's weight may be offset by the centrifugal force due to the curved path around the earth, the dynamic pressure is less than at slower speeds so the frequency of all oscillatory motions is correspondingly lower.

The angular motion of a skip rocket during the first part of the skip when the vehicle is approaching the earth resembles the behavior noted for ballistic vehicles. During the second part of a skip when the vehicle leaves the atmosphere some interesting motions are possible and the behavior during a complete skip for a typical vehicle is thus worth some discussion. As an example, consider a skip rocket which consists essentially of a triangular plan-form wing with large leading-edge-sweep angle and with a root-chord length of about 50 feet. For an assumed wing loading of about 20 pounds per square foot, a lift-drag ratio of 6 and a flight range of about 4000 nautical miles, the speed on entering the atmosphere will be 14,500 feet per second before the first skip at a flight path angle, θ, of about 12° below the horizontal. The path during the first skip is shown in the lower part of Fig. 16. Suppose on the approach to the earth the vehicle has no angular velocity but is pitched an angle α_E away from the correct trim angle. If the static margin is 5 percent of root chord, then the initial angular response to the increasing dynamic pressure, which, as with the ballistic vehicle, is mathematically expressible as a Bessel function of the first kind (zero order), is that shown on the left in the upper part of Fig. 16. The amplitude of the oscillation diminishes and frequency increases as the bottom of the skip is approached. The maximum frequency reached is slightly less than one cycle per second. If the dynamic stability were zero the motion would diverge as the vehicle left the atmosphere. Now, curiously enough, while the motion during the outgoing flight could be represented by the reverse of the same Bessel function, it might not be the same since the general solution is of the form

$$\alpha = C_1 J_0(-) + C_2 Y_0(-)$$

and it is possible that C_1 may be zero rather than C_2. In the event that this is the case, the Bessel function of the second kind (zero order) gives the motion shown on the right in the upper part of the figure; that is, the vehicle is left with an angular velocity as it leaves the atmosphere. This tumbling would not present a serious problem since the angular rate is only one-tenth of a revolution per minute which could be checked readily by reaction controls. Fortunately, it appears that the dynamic stability would actually be so stabilizing for the case assumed as to well damp the motion as the bottom of the skip was approached, as is also indicated in the figure.

In the flight of a glide rocket the flight path at rocket burnout may accidentally be higher or lower than the equilibrium altitude or the flight path angle may accidentally be higher or lower than that required to follow the equilibrium trajectory. In either event the vehicle will follow a skip path before recovery to the equilibrium path may be effected. In this event it follows, from what has been said about the skip-rocket stability problem, that if the glide rocket oscillates, it will do so at lower frequency than it would at equilibrium altitude when above this altitude, and at higher frequency when below it. The amplitude variation would, of course, be the opposite of this behavior.

Comparison of Flight Efficiencies

In the section on performance it was pointed out that the measure of efficiency for long-range flight of the three types of rockets and of the supersonic airplane would be chosen as the ratio of the initial weight to payload. It is in order, now, in the light of the discussion of aerodynamic heating, loading, and stability, to adjudge the relative efficiencies of these vehicles. To review, on the basis of the ratios of initial to final weight given in Fig. 8(a), it was noted that for ranges less than half the circumference of the earth, the supersonic airplane was the most attractive and the ballistic vehicle the least. The glide and skip rockets are not only intermediate in this regard but have approximately equal weight ratios.

On the other hand, for the ballistic vehicle the payload will generally be a larger fraction of the final weight than for the other vehicles since

1. By using a high drag shape for the re-entry body the coolant weight to protect it from aerodynamic heating will generally be small.

2. The re-entry body is relatively small and robust so that its structural weight should be low in spite of the large drag and side forces that may be experienced.

3. If, then, the payload is a large fraction of the re-entry body weight, the propellant tankage weight, which is directly geared to this weight, can be kept small through clever design.

4. The motor weight, as for all the rocket craft, will be low (see Fig. 1).

For the glide rocket in comparison with the airplane the ratio of payload to final weight has a chance of being superior by virtue of the engine weight advantage. However, it was noted that low wing loading appears to be a necessity for these vehicles if little or no coolant is to be required, and thus this rocketcraft does not have the natural robustness of the ballistic re-entry body. Rather, the structural problem is more nearly that for the conventional airplane with the exception that the surface, at least, must be able to withstand high temperatures. Success or failure of glide rockets is clearly to be determined by the state of the metallurgical and structural arts and the ingenuity of the designer.

The skip rocket is certainly unattractive for the present at least. Its coolant requirements are severe. Moreover, the structural loads are fairly large and occur when aerodynamic heating is intense. Thus, it is to be expected that the disadvantage of high coolant and structural weight will far more than offset the very marginal advantage it has as regards initial-to-final-weight ratio when compared with the glide rocket.

In short, it is my opinion that the ballistic vehicle can compare very favorably with the supersonic airplane for long range as well as short range flight and the glide rocket may also prove to be attractive, but not so the skip rocket.

New Problems of Hypervelocity Vehicles

Up to this point the discussion of the problems of rocket vehicles has been confined to the effects of phenomena which have in the past been important ones for lower speed aircraft and will continue to be important for aircraft of all speeds. Now with considerable extension of both speed and altitude, other phenomena also become important. The nature of some problems will be altered, as a result, and new problems, of course, will be encountered.

First, it is well to note that our interest in bluff bodies for ballistic vehicles in particular, and in rounded-nosed bodies generally, has changed our emphasis in aerodynamics. The detached bow waves which occur with such bodies at high supersonic speeds complicate the calculations of the flow-field characteristics. In the present period, much attention is being given to such studies.[14] In addition, at the very high altitudes attained by most of the rocketcraft, the mean-free-path of air molecules can be of the same order, or long, compared to the dimensions of the vehicles. Thus, slip-flow and free-molecule-flow studies are of interest, particularly for satellite vehicles.[10] The aerodynamicist must deal with air having unfamiliar states and properties.

Second, at hypersonic speeds where, for example, air is greatly decelerated it may undergo considerable change in composition,[15] the degree depending upon many factors. Dissociation of oxygen and nitrogen molecules can occur and, in addition, thermal ionization of many of the constituents. It is naturally to be expected that the convective heat transfer will, as a result, be altered from what it was for the "perfect" gas, and this has been the subject of much recent research effort.[16] Moreover, the decelerated gas becomes capable of radiating energy and the radiative heat transfer must generally be considered for hypersonic vehicles, particularly for long-range ballistic rockets. It is not only the aerodynamic heating problems that are affected. The fact that at very high air temperature the gas becomes electrically conductive[16] introduces new problems in radio wave transmission and reception. In addition, a conducting gas flow can, of course, be influenced by a magnetic field. The study of such flows, which has been termed "magneto gas dynamics," is still in too primitive a state to indicate how important a role it can play, but many interesting possibilities suggest themselves.[17]

Third, our experience with airplanes powered by air-breathing engines has naturally been restricted to the stratosphere, or lower. Our ignorance increases with altitude. For rockets, literally, "the sky's the limit," and it is not surprising that a great emphasis has now been placed on obtaining a more thorough understanding of the whole atmosphere.[10] These studies are not aimed at an understanding of the chemical and physical characteristics alone, but also of the occurrence of high-energy particles, from meteors to cosmic rays, and the nature of the problems they will promote.

This discussion of new problems has only touched upon a few of the known phenomena which become of interest in consideration of high-speed rockets. Certainly numerous others will appear as the conquest of space progresses. Faced with such a nebulous state of affairs it is not surprising that our approach to these new problems is a cautious one. It is well to note, however, that in this regard the present situation is certainly analogous to that which the Wright brothers faced at the turn of the century. If we give the same painstaking and intelligent treatment to our problems as they gave to theirs a half century ago, our success seems assured.

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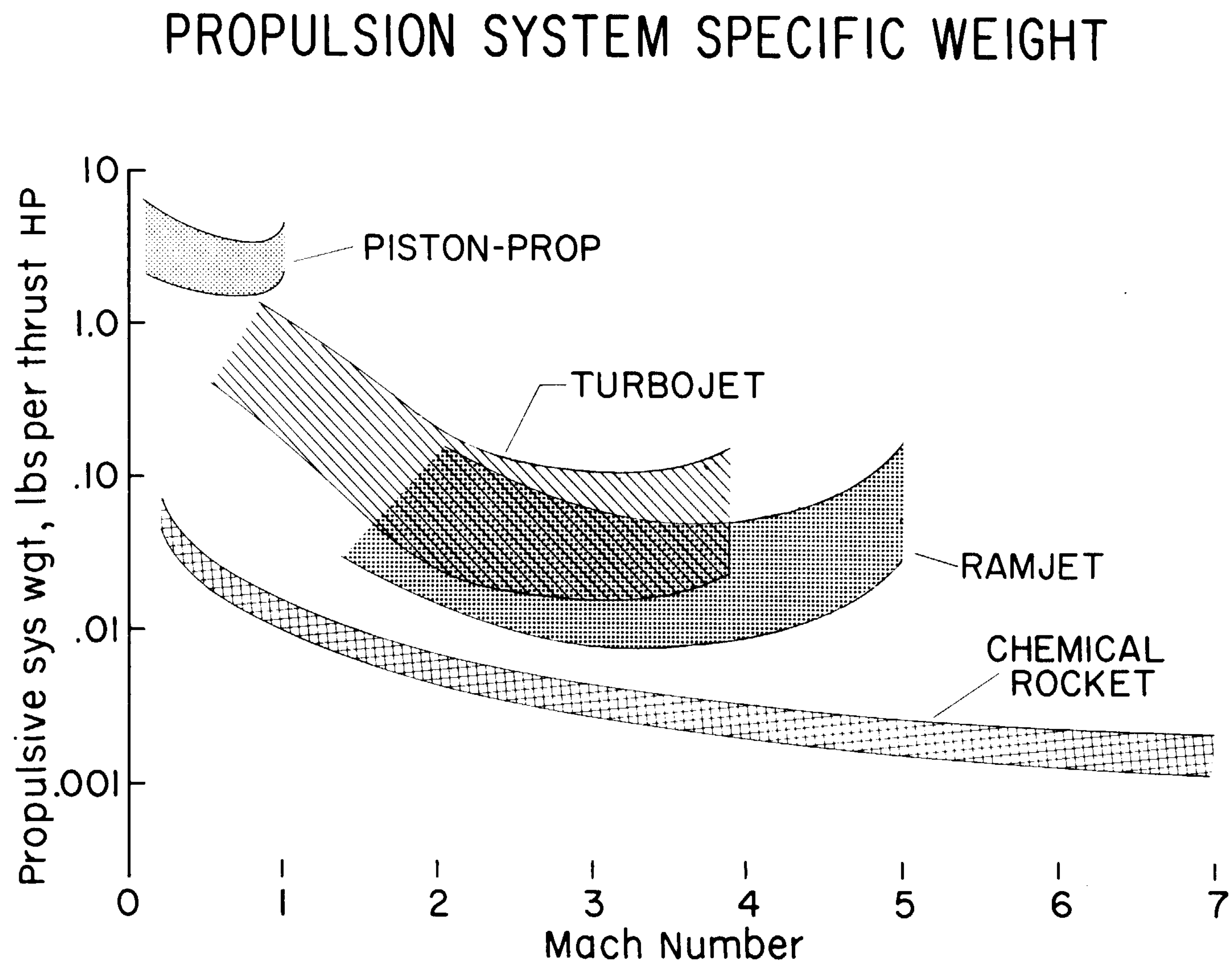

PROPULSION SYSTEM SPECIFIC WEIGHT
Propulsive sys wgt, lbs per thrust HP
10
1.0
.10
.01
.001
PISTON-PROP
TURBOJET
RAMJET
CHEMICAL
ROCKET
0
1
2
3
4
5
6
7
Mach Number

Figure 1

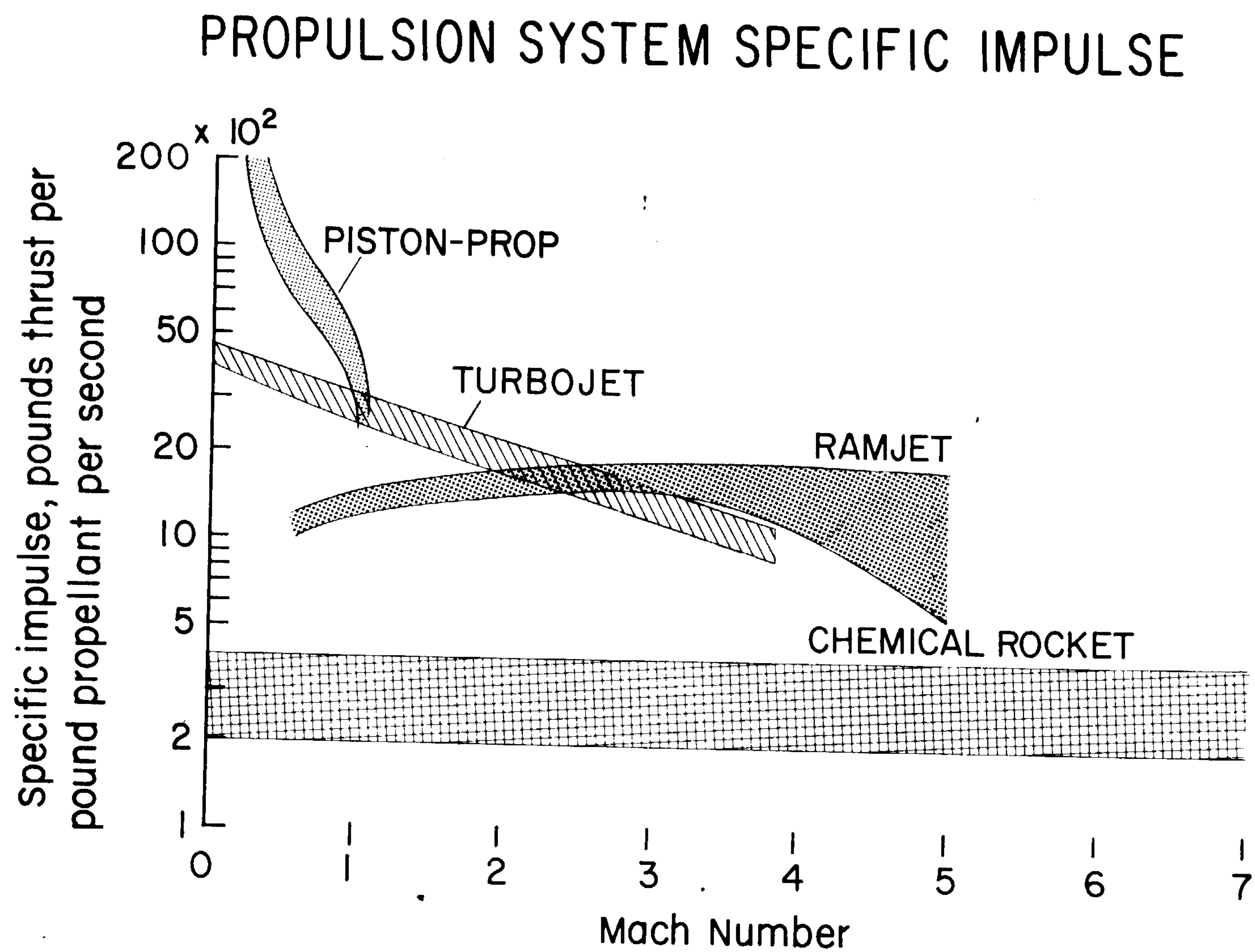

PROPULSION SYSTEM SPECIFIC IMPULSE
Specific impulse, pounds thrust per pound propellant per second
200
x 10²
100
50
20
10
5
2
1
PISTON-PROP
TURBOJET
RAMJET
CHEMICAL ROCKET
0
1
2
3
4
5
6
7
Mach Number

Figure 2

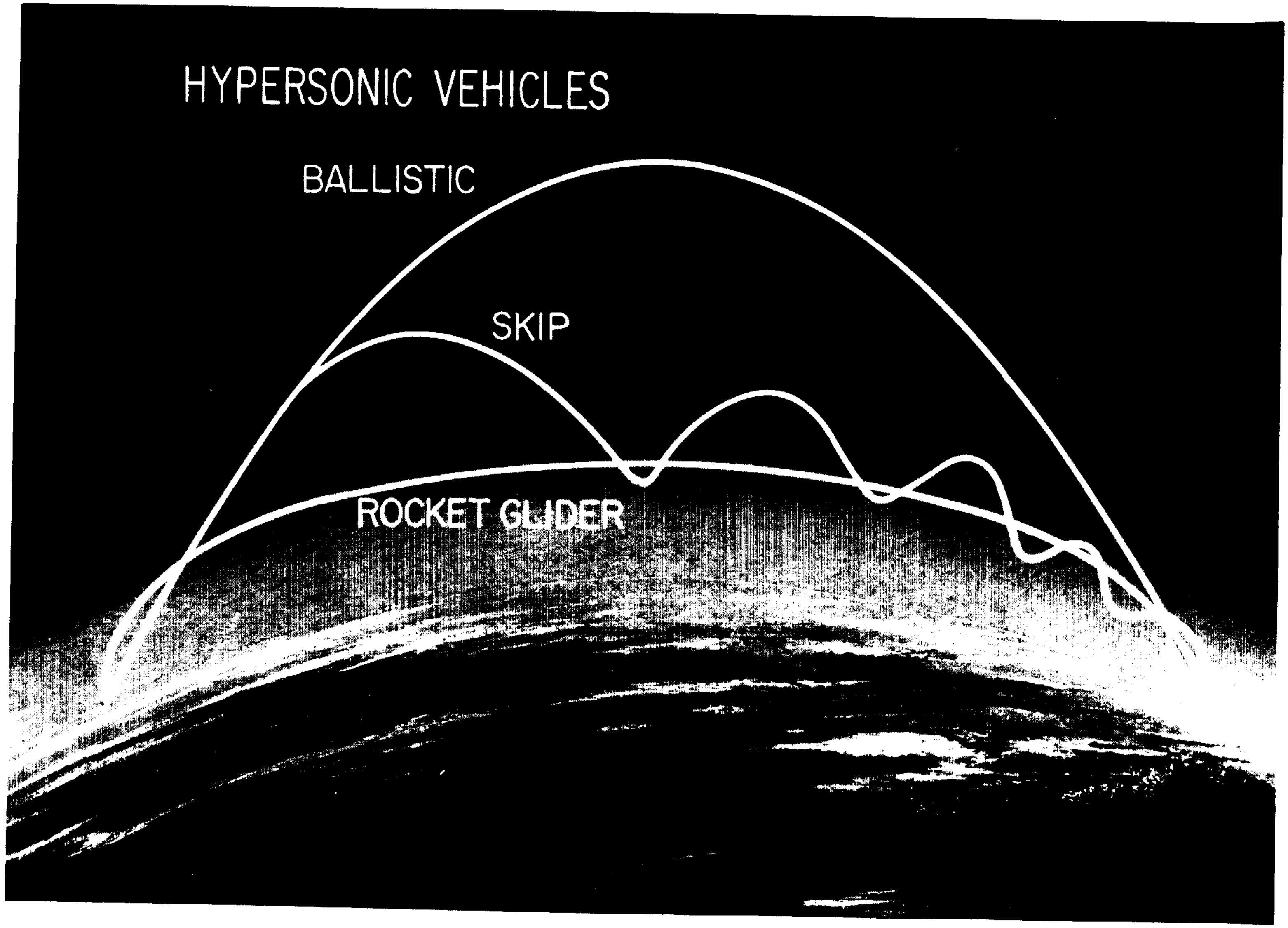
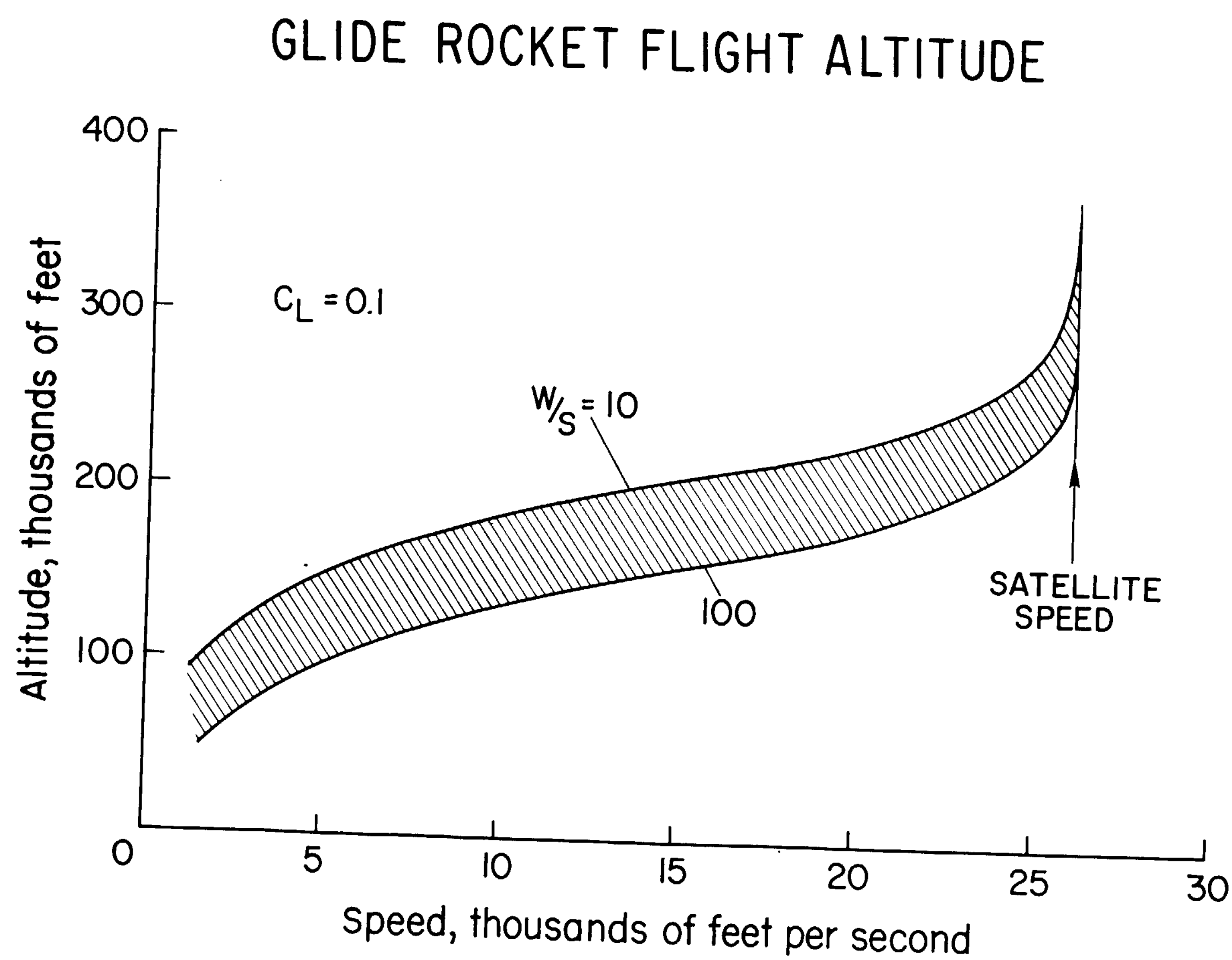


Figure 3



GLIDE ROCKET FLIGHT ALTITUDE
400
300
200
100
0
Altitude, thousands of feet
C_L = 0.1
W/S = 10
100
SATELLITE SPEED
5
10
15
20
25
30
Speed, thousands of feet per second

Figure 4

TRAJECTORY ANGLES AT END OF ROCKET BOOST

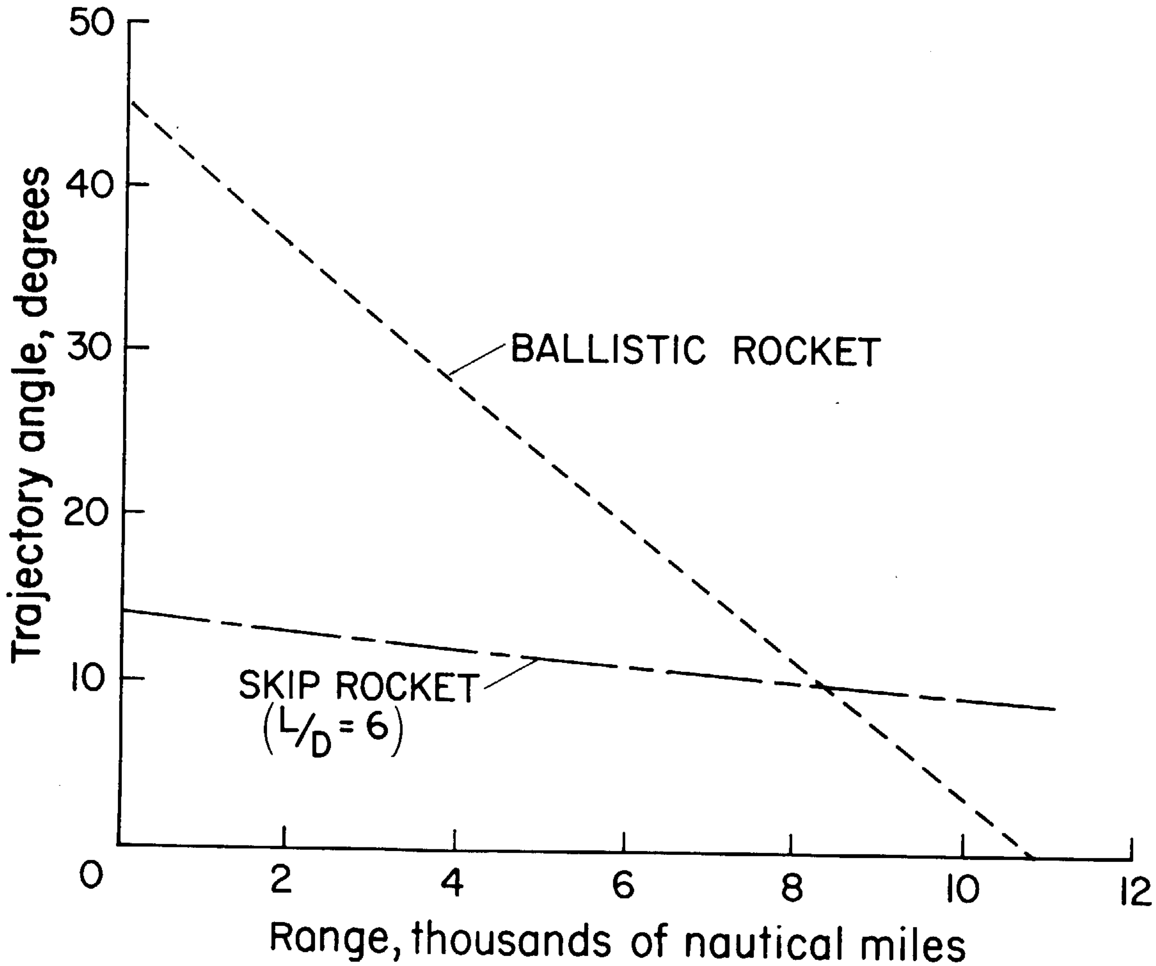


Figure 5.

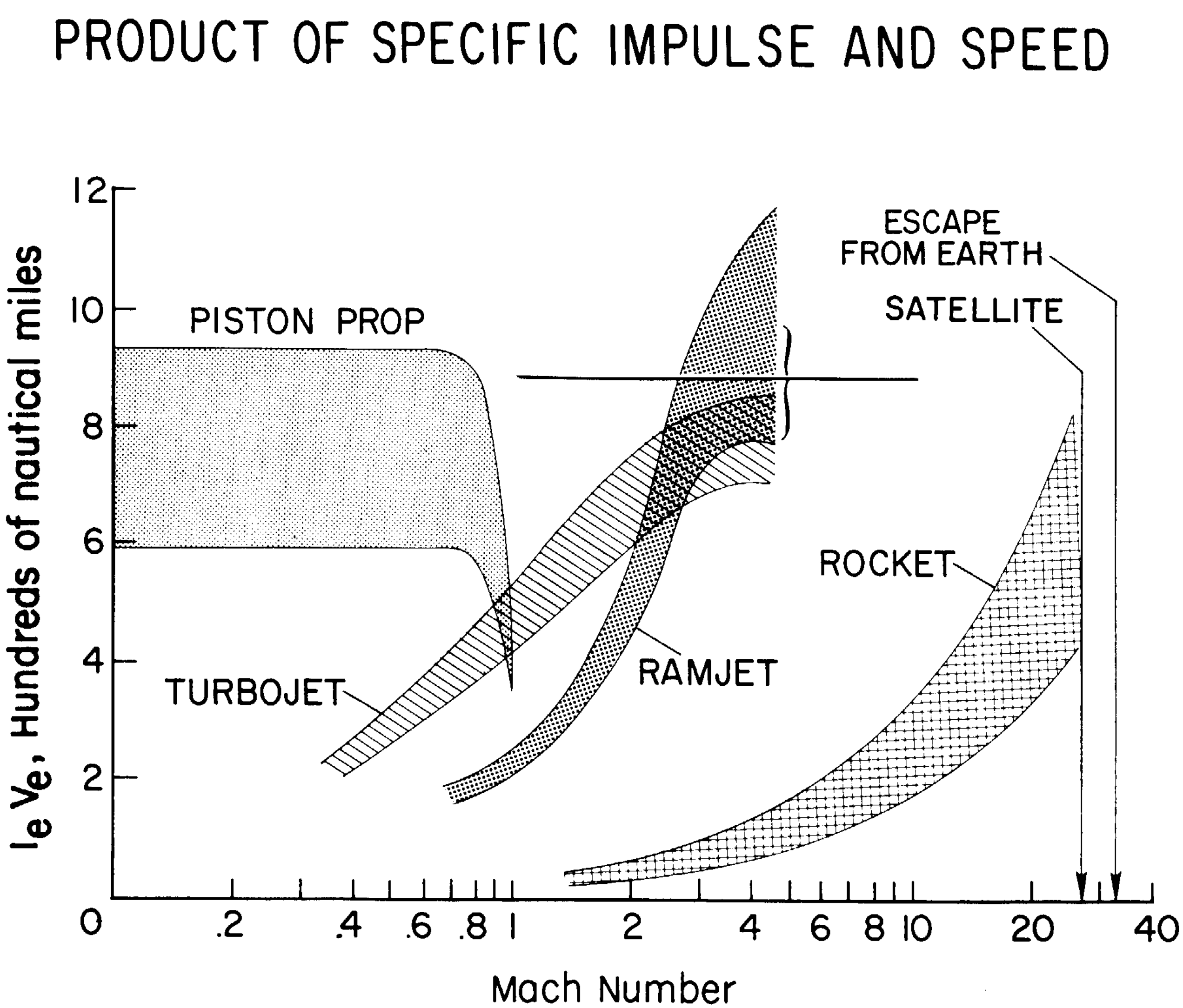

PRODUCT OF SPECIFIC IMPULSE AND SPEED
$I_e V_e$, Hundreds of nautical miles
12
10
8
6
4
2
0
.2
.4
.6
.8
1
2
4
6
8
10
20
40
Mach Number
PISTON PROP
TURBOJET
RAMJET
ROCKET
ESCAPE FROM EARTH
SATELLITE

Figure 6.

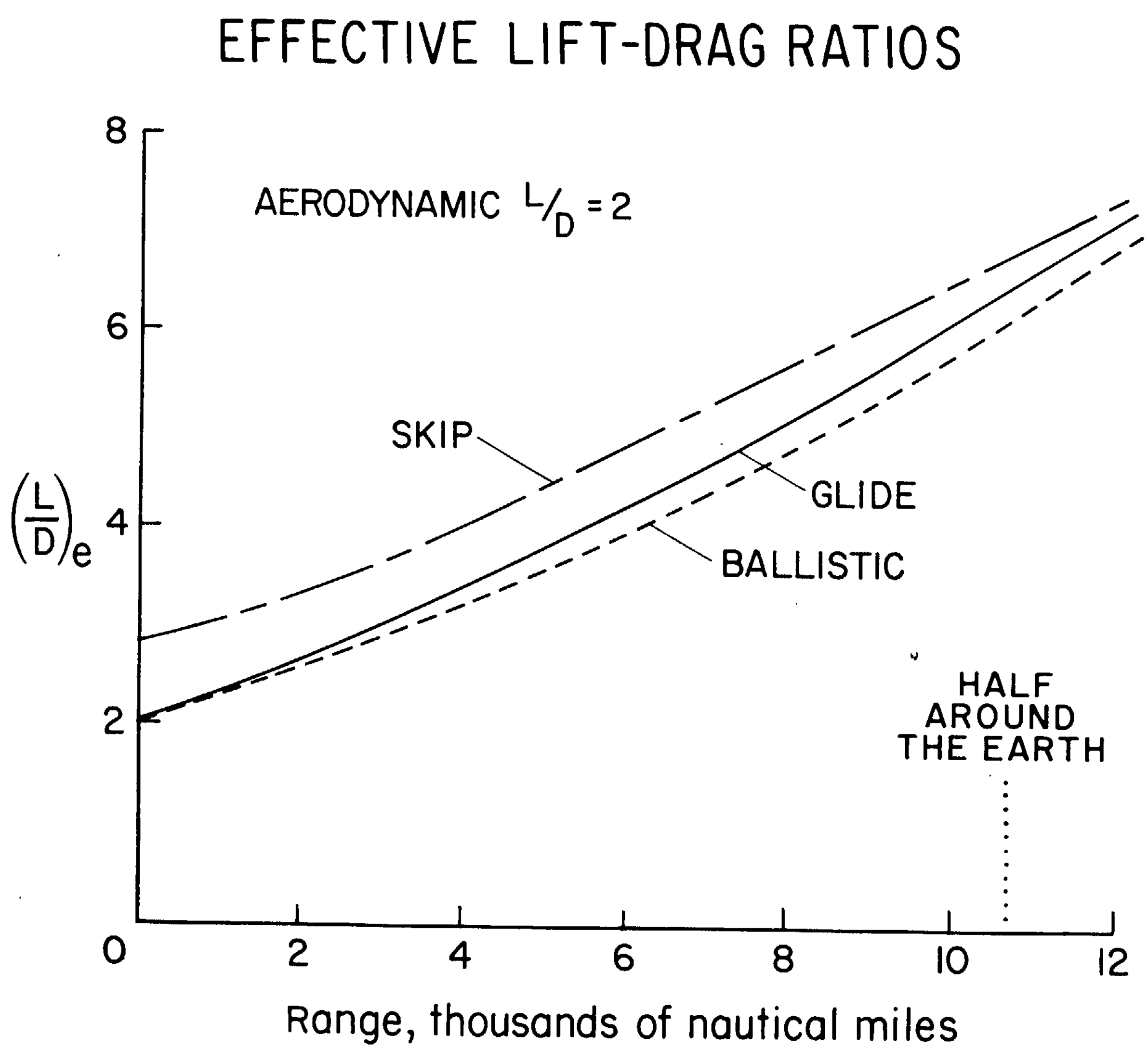

EFFECTIVE LIFT-DRAG RATIOS
AERODYNAMIC L/D = 2
SKIP
GLIDE
BALLISTIC
HALF AROUND THE EARTH
(L/D)e
0
2
4
6
8
10
12
Range, thousands of nautical miles

Figure 7(a).

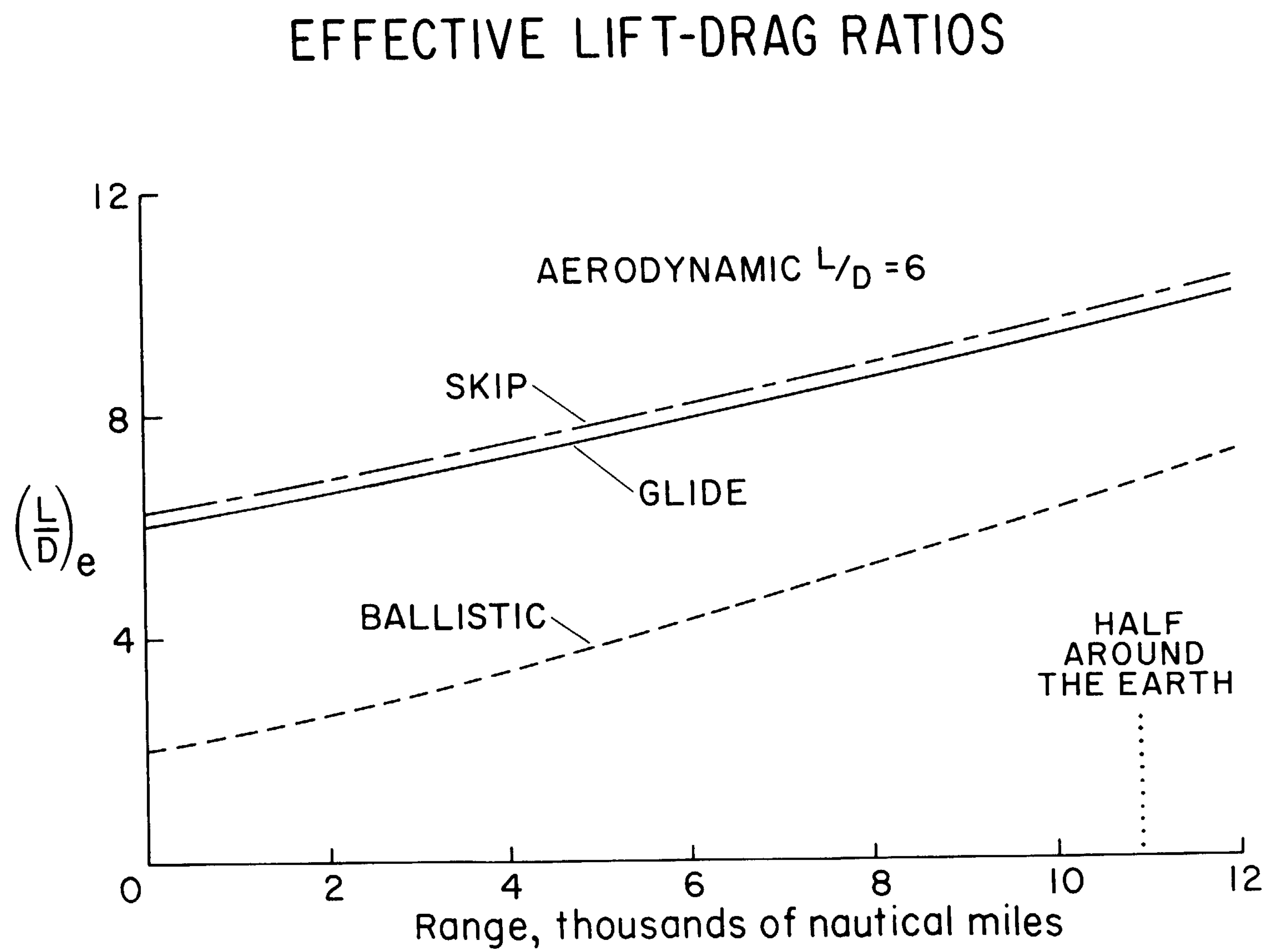


Figure 7(b).

ORIGINAL PAGE IS
OF POOR QUALITY

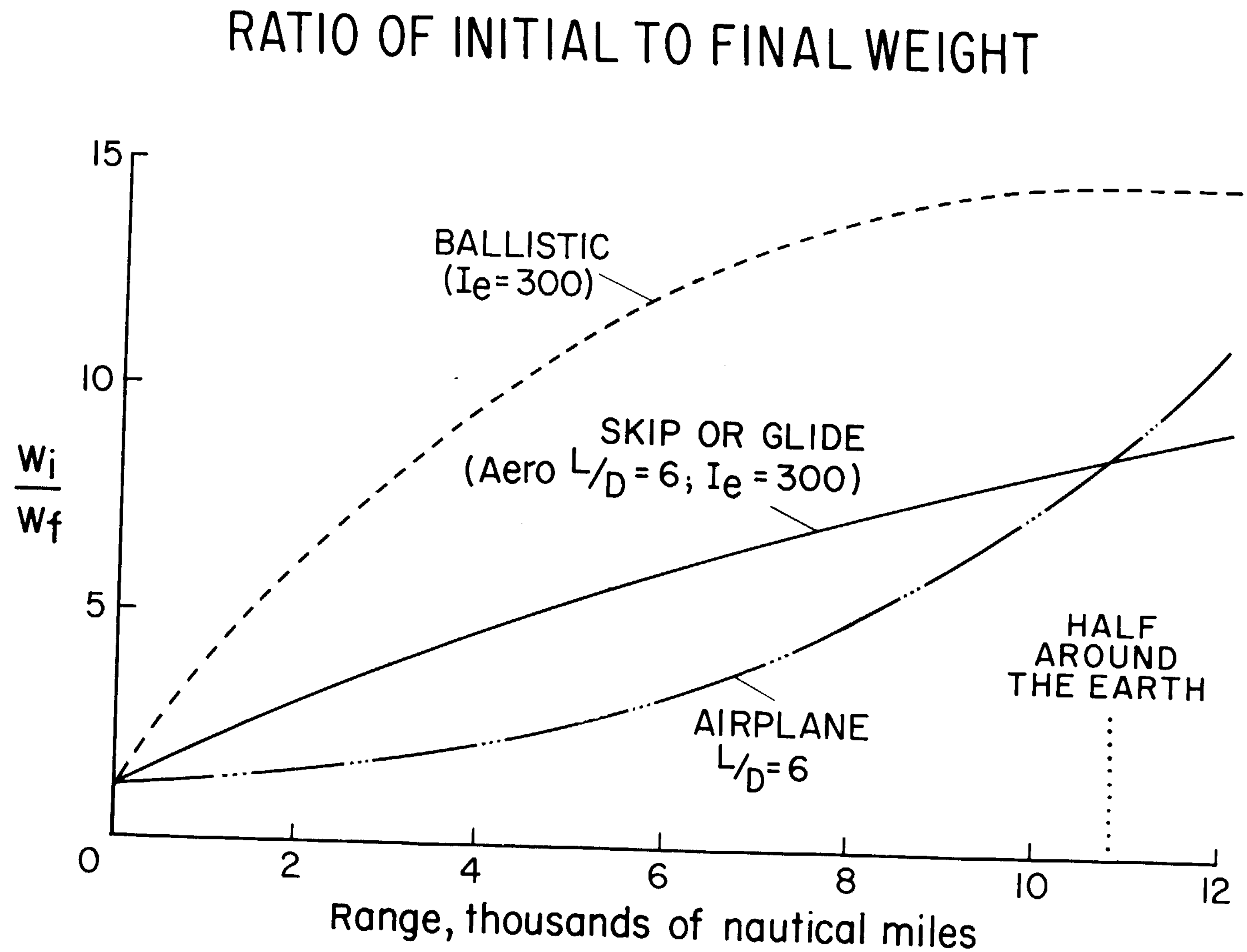

RATIO OF INITIAL TO FINAL WEIGHT
15
10
5
0
$\frac{W_i}{W_f}$
BALLISTIC
(I_e=300)
SKIP OR GLIDE
(Aero L/D = 6; I_e = 300)
AIRPLANE
L/D=6
HALF
AROUND
THE EARTH
2
4
6
8
10
12
Range, thousands of nautical miles

Figure 8(a).

RATIO OF INITIAL TO FINAL WEIGHT

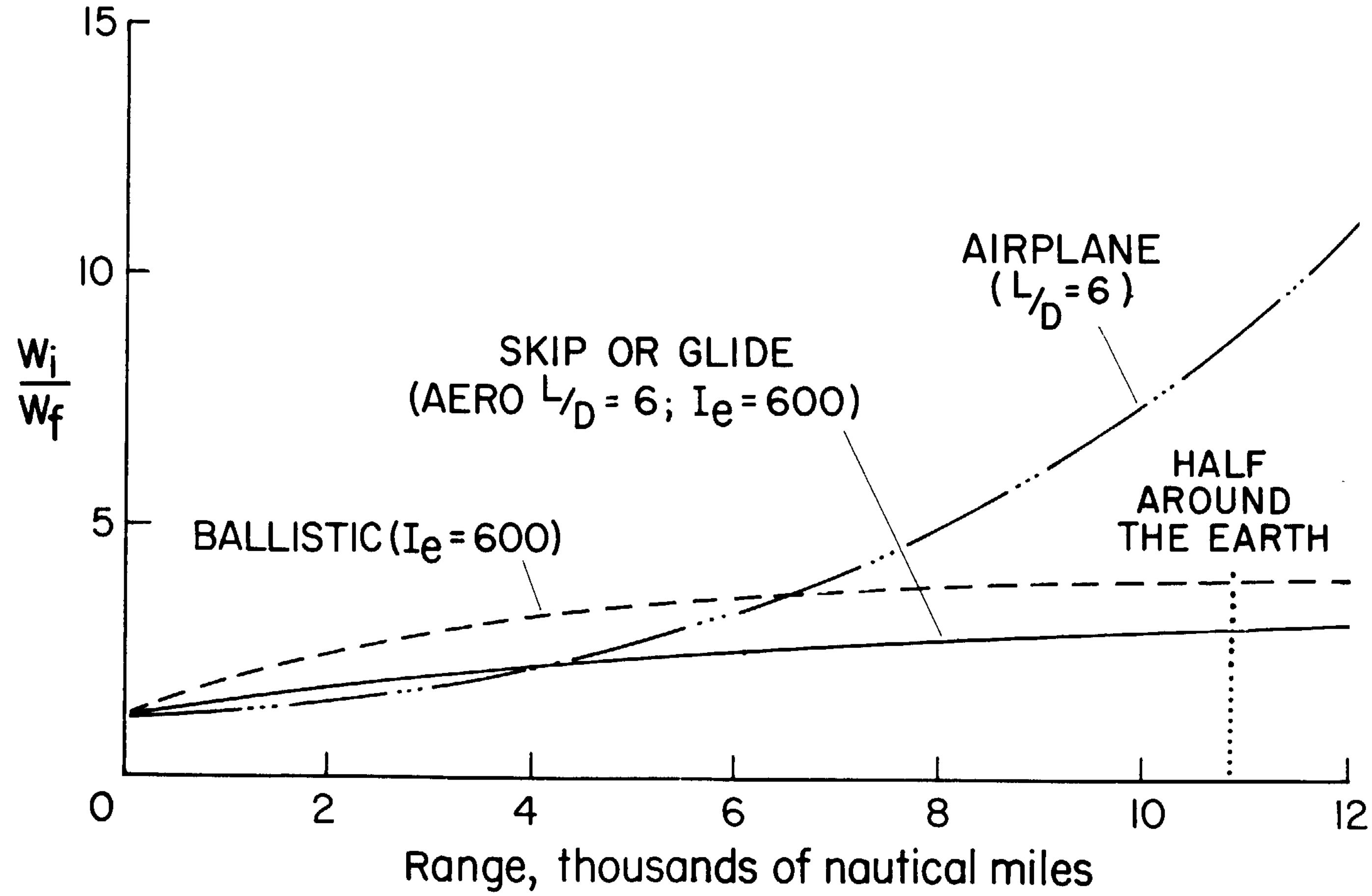


Figure 8(b).

MAXIMUM FLIGHT SPEEDS

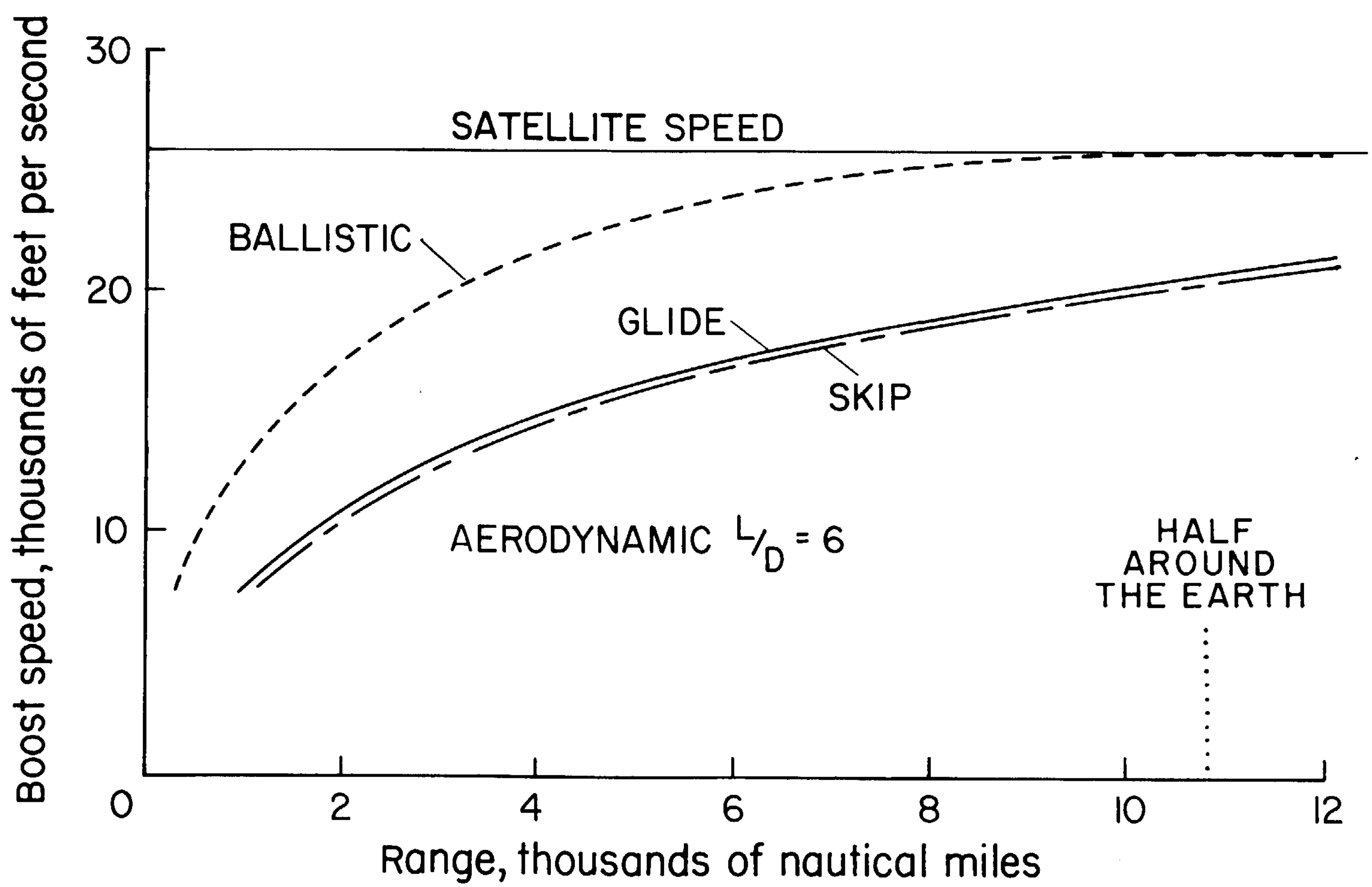


Figure 9.

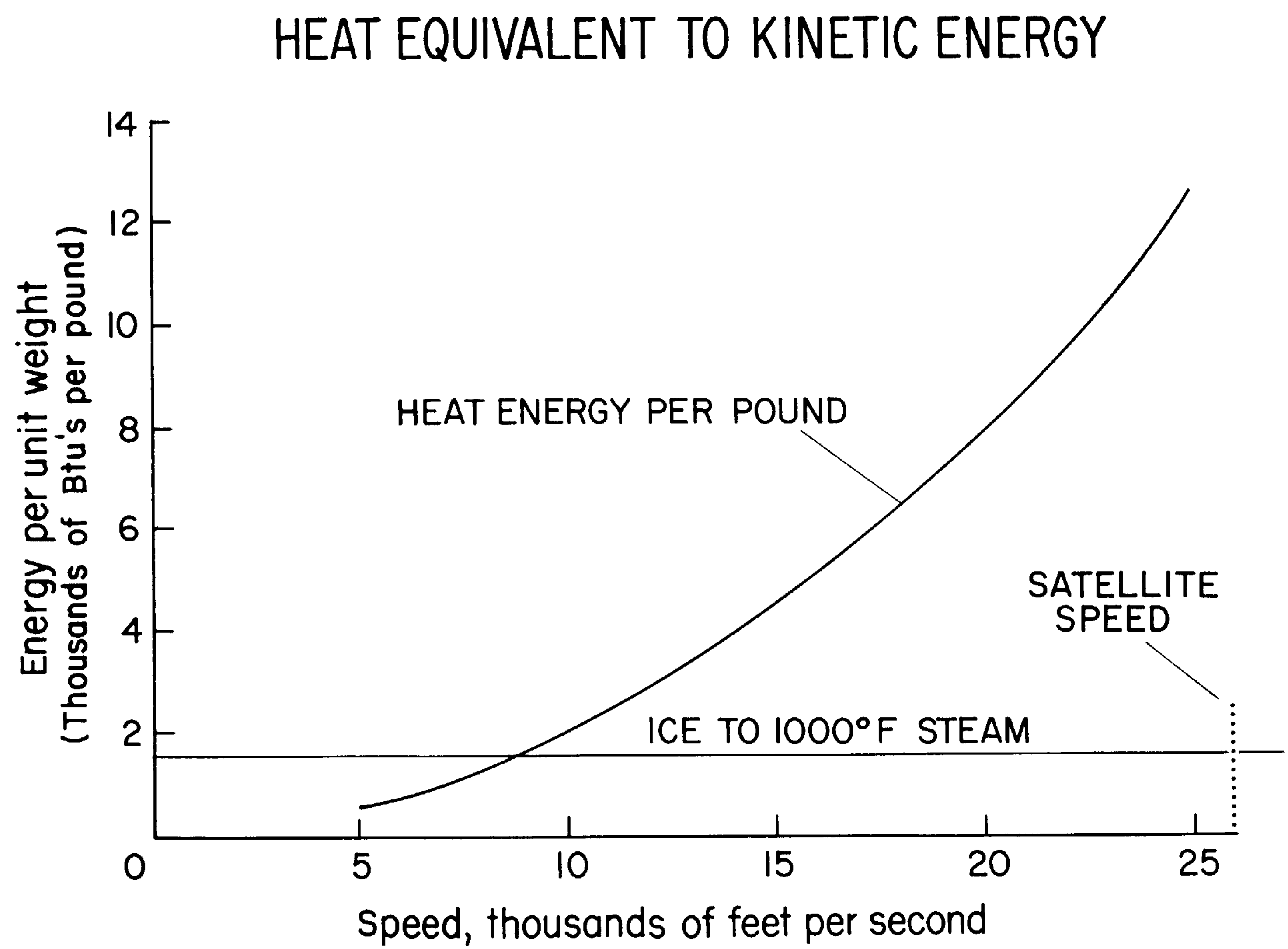

HEAT EQUIVALENT TO KINETIC ENERGY
Energy per unit weight
(Thousands of Btu's per pound)
14
12
10
8
6
4
2
0
5
10
15
20
25
HEAT ENERGY PER POUND
SATELLITE SPEED
ICE TO 1000°F STEAM
Speed, thousands of feet per second

Figure 10.

RADIATION EQUILIBRIUM TEMPERATURES

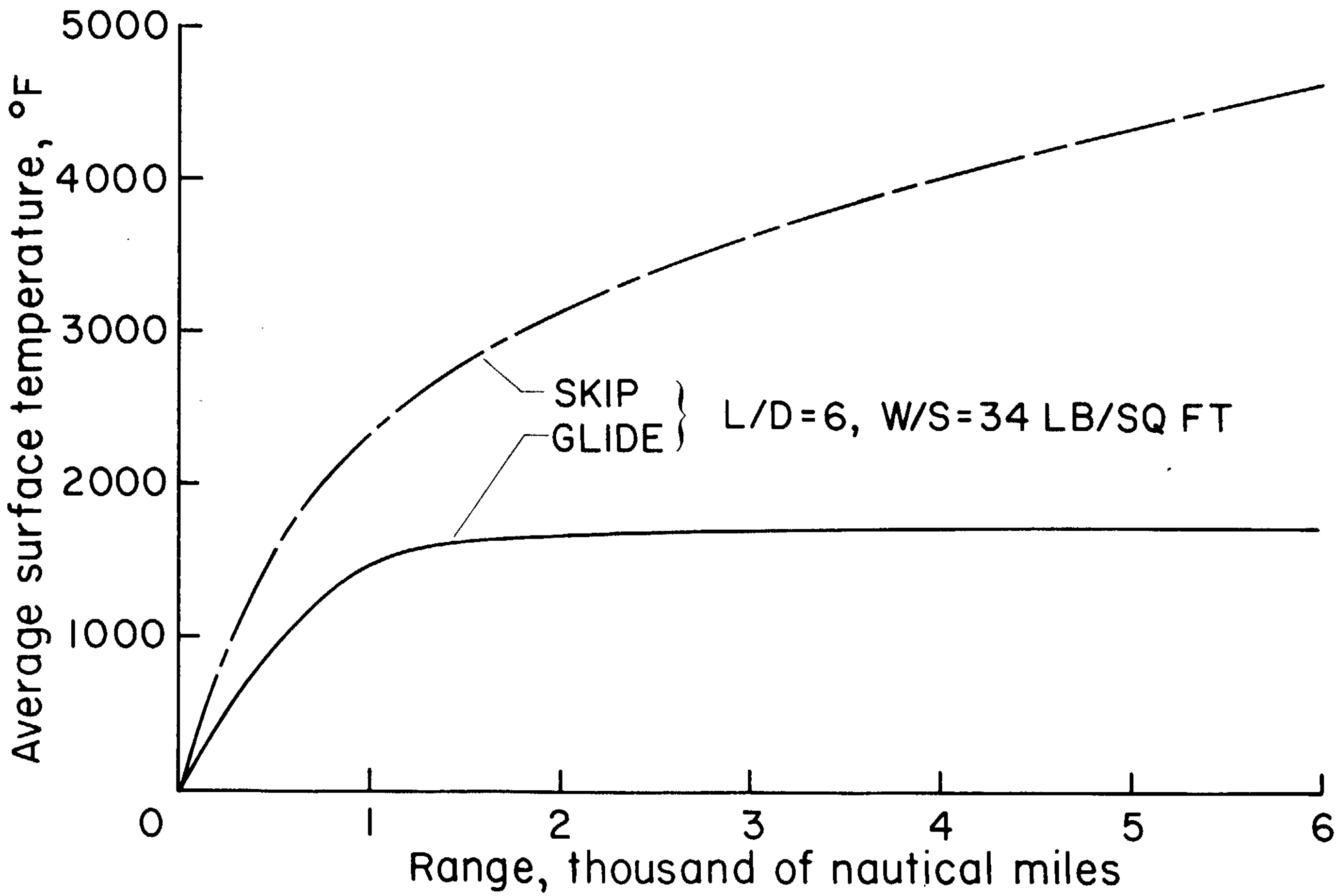


Figure 11.

EFFECT OF SIZE ON DECELERATION OF IRON SPHERES

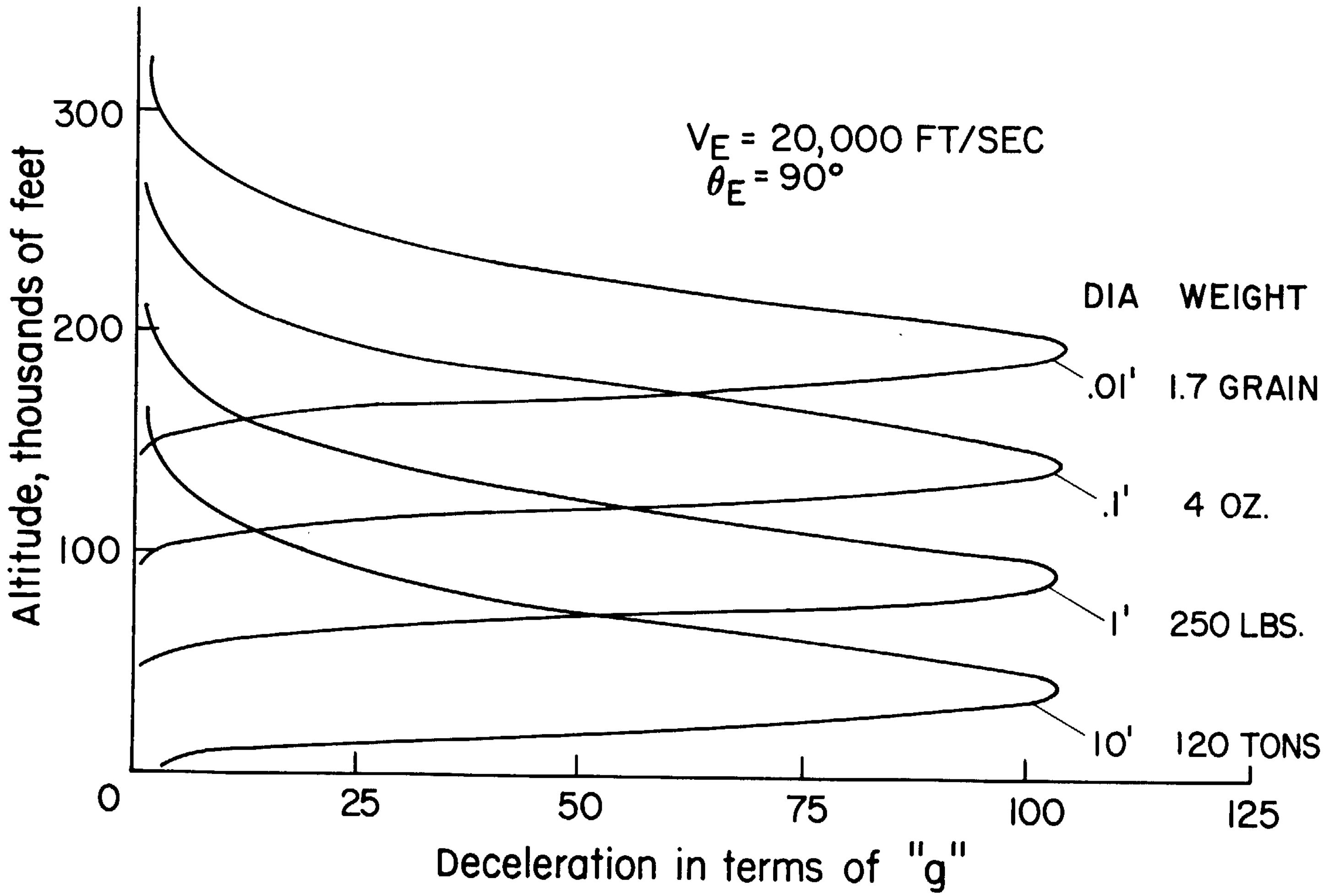


Figure 12.

MAXIMUM DECELERATION FOR MINIMUM ENERGY FLIGHT OF BALLISTIC VEHICLE

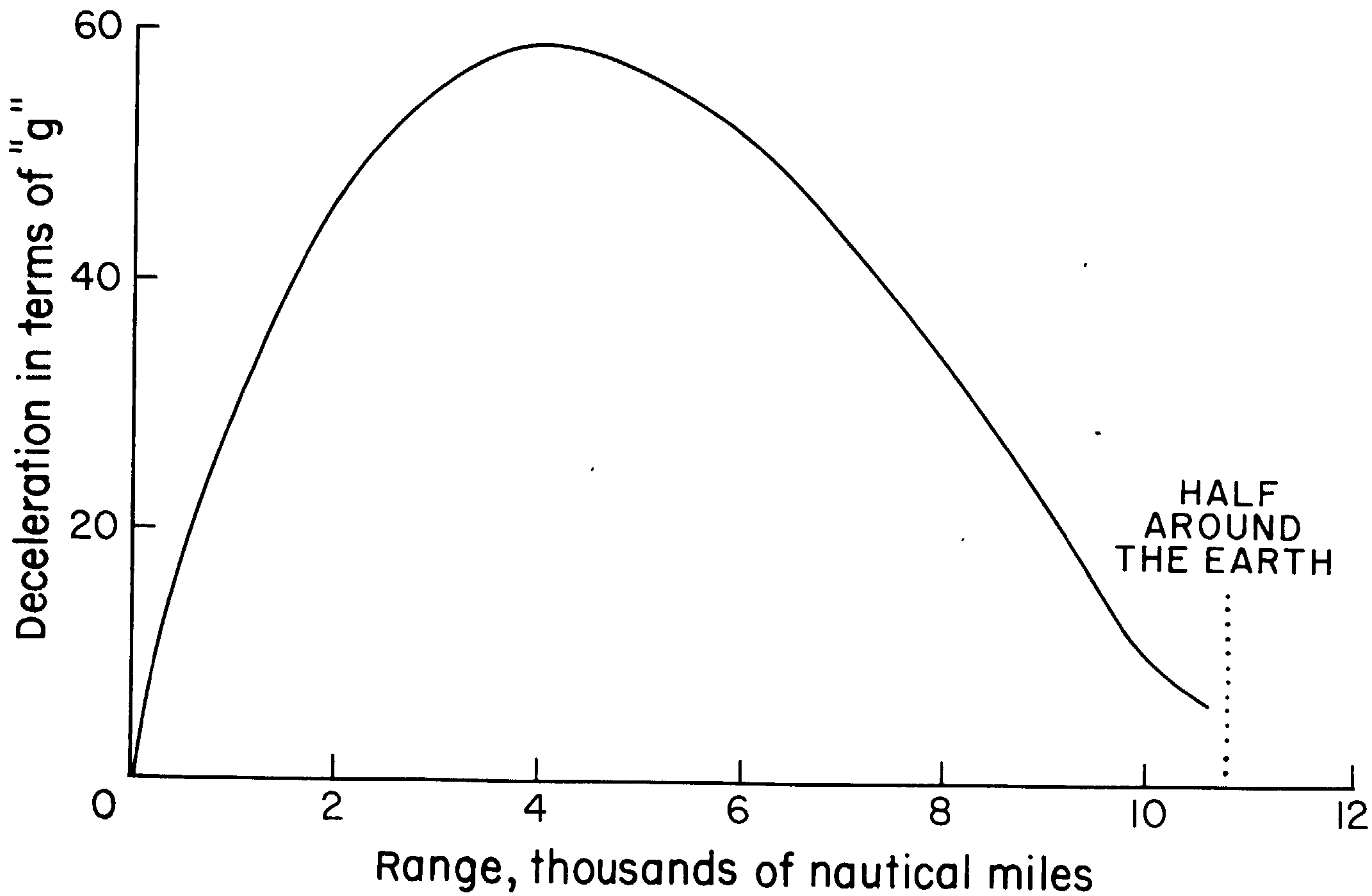


Figure 13.

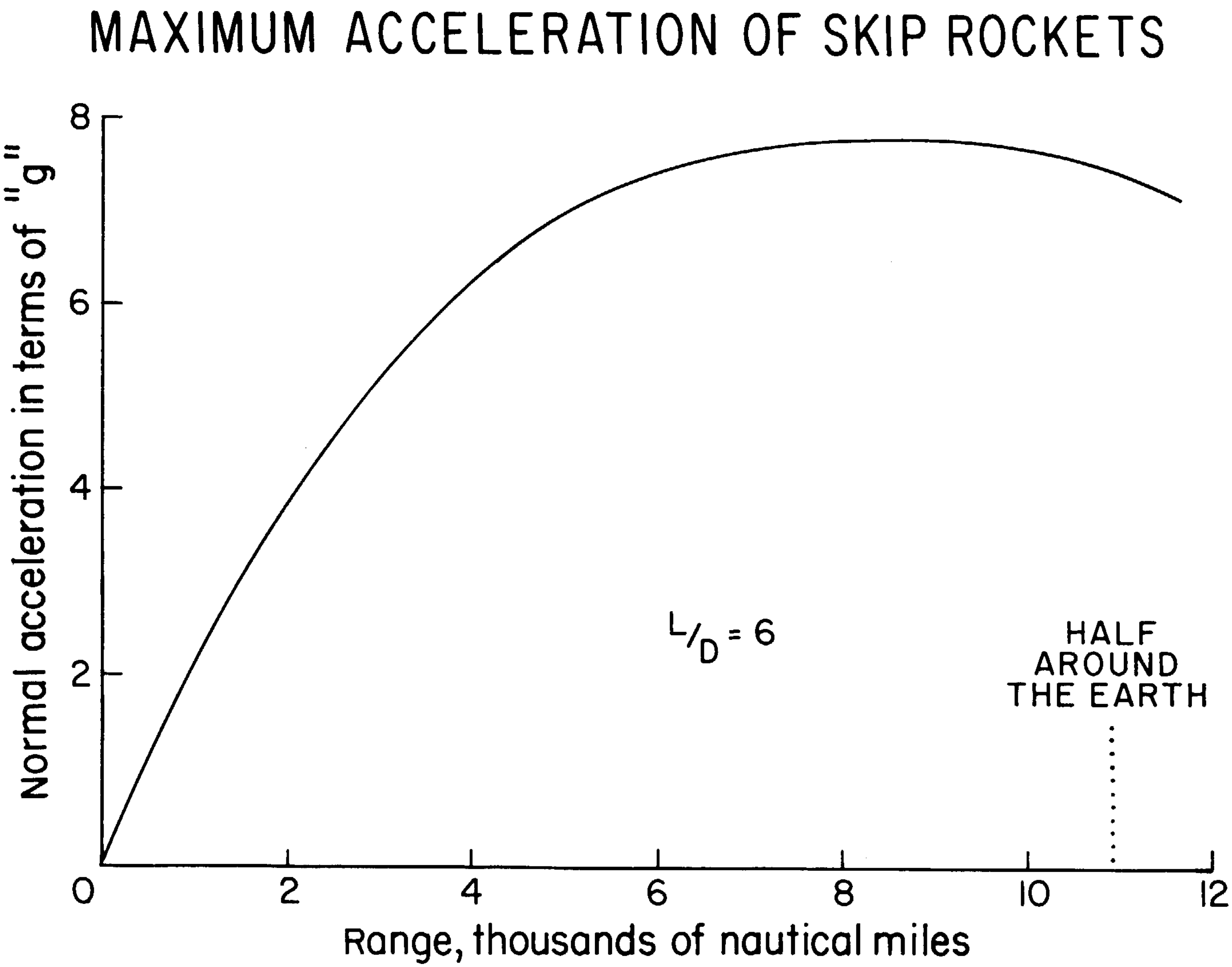

MAXIMUM ACCELERATION OF SKIP ROCKETS
Normal acceleration in terms of "g"
0
2
4
6
8
10
12
Range, thousands of nautical miles
L/D = 6
HALF
AROUND
THE EARTH

Figure 14.

OSCILLATORY MOTION OF A BALLISTIC VEHICLE

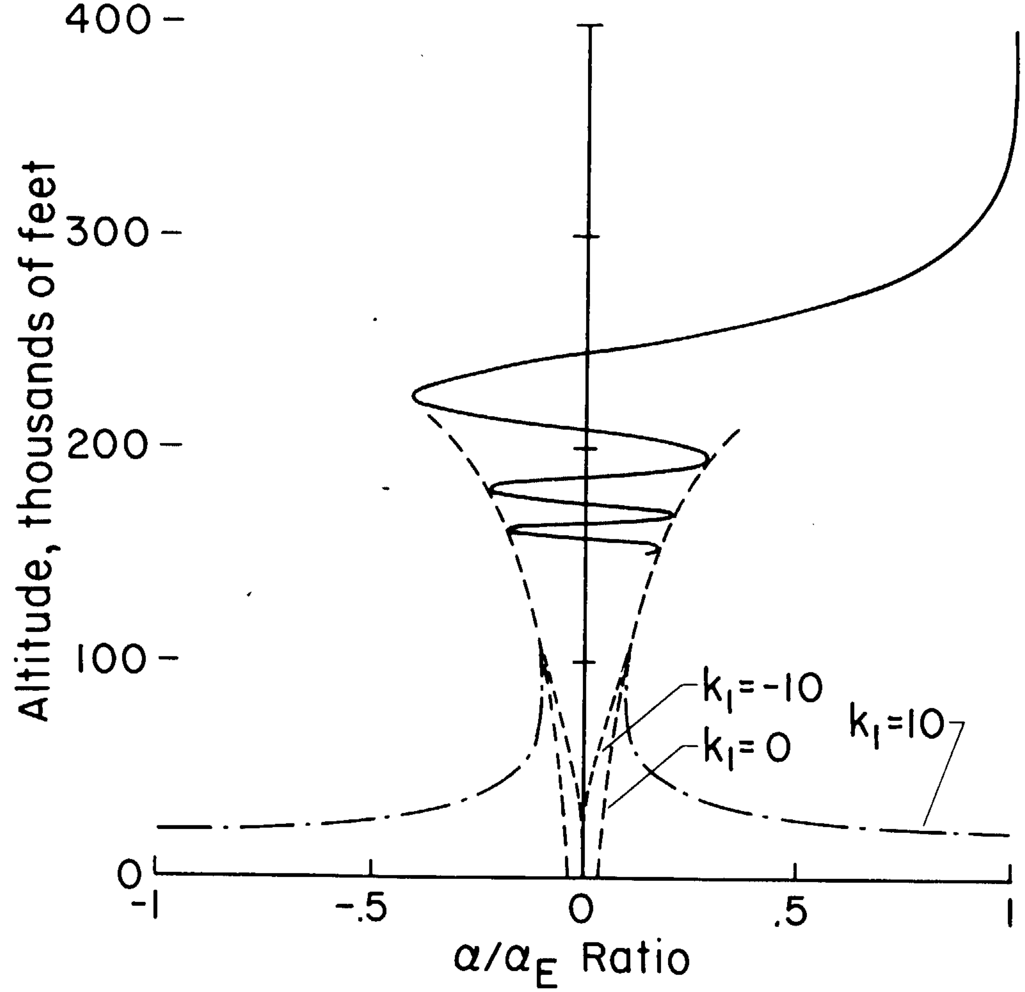


Figure 15.

OSCILLATORY MOTION OF A SKIP ROCKET

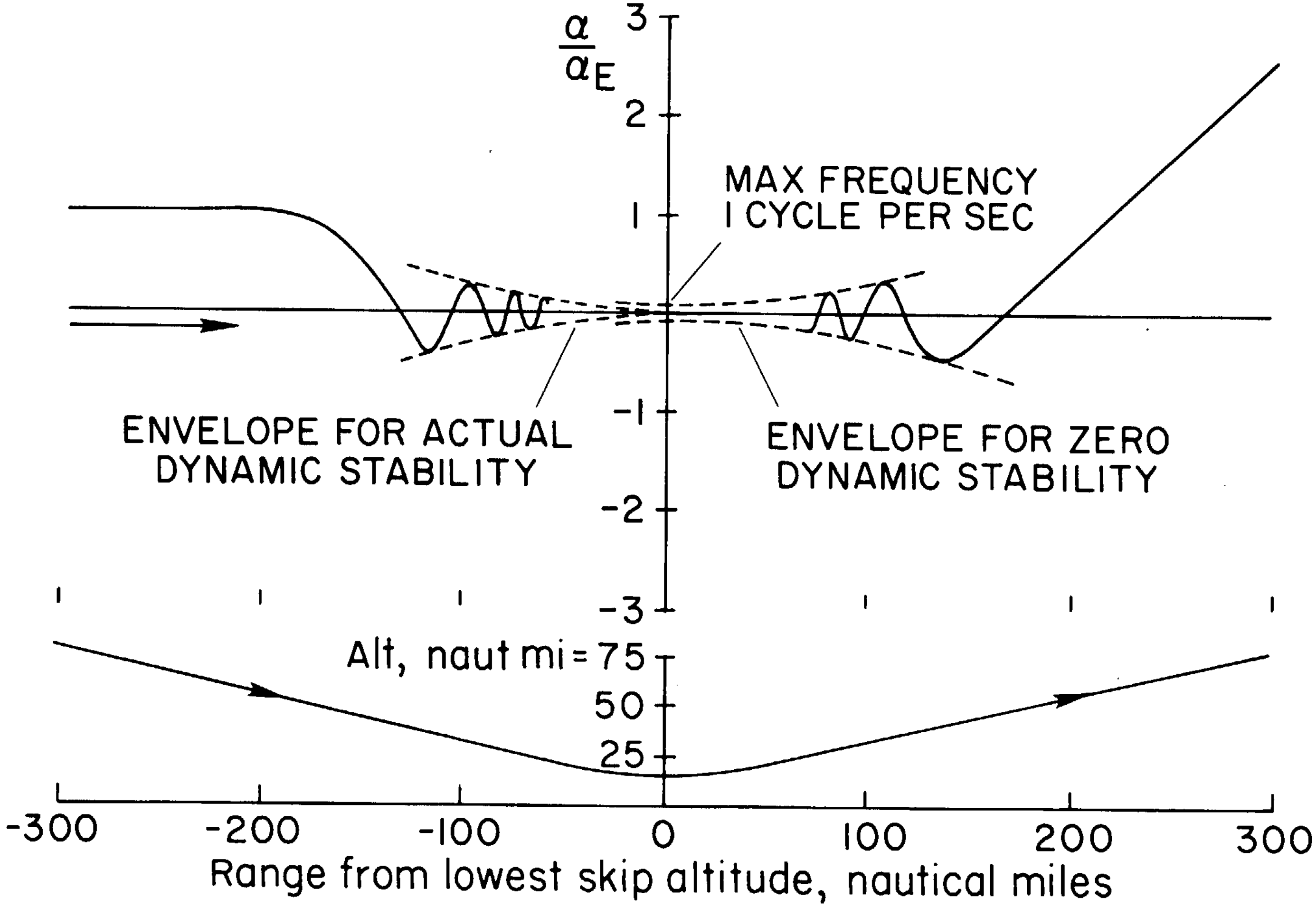


Figure 16.